(Pages : 4) C – 1436

Reg. No. : .....

Name : .....

Sixth Semester B.Sc. Degree Examination, April 2017
First Degree Programme Under CBCSS
MATHEMATICS
Core Course X
MM 1642 : Linear Algebra
(2014 Admn.)

Time : 3 Hours Max. Marks : 80

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SECTION – I

All the first **10** questions are **compulsory**. **Each** question carries **1** mark.

1. Write an equation for the line through the vector $A = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$.
2. Find a vector U such that the line with equation $y = 2x$ lies along U.
3. Find the angle between the vectors $\begin{pmatrix} 4 \\ 4 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 5 \end{pmatrix}$.
4. Find the distance from the $\begin{pmatrix} 1 \\ 4 \end{pmatrix}$ to the line along the vector $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$.
5. If R_θ denotes the rotation by θ radians, find $R_{\frac{3\pi}{2}}\left(\begin{pmatrix} x \\ y \end{pmatrix}\right)$.
6. Define linear transformation of the plane.
7. Let L be a straight line through the origin. Find the matrix of the transformation S which reflects each vector in L.
8. Find the characteristic equation of the matrix $\begin{pmatrix} 3 & -2 \\ 9 & -3 \end{pmatrix}$.

P.T.O.

9. Prove that the only eigenvalues of an isometry of R^3 are 1 or – 1.

10. Classify the quadric surface $x^2 + 4y^2 + z^2 - 2x = 0$.

SECTION – II

Answer **any 8** questions from this Section. **Each** question carries **2** marks.

11. Let K be reflection in the x_1x_2 – plane and J be reflection in the x_2x_3 – plane. Find JK.

12. If $m(T) = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \end{pmatrix}$ find $m(T^{-1})$.

13. Let A be a linear transformation and let L be as straight line. Prove that the image of L under A is either a straight line or a single point.

14. Let T_1 and T_2 be two length preserving transformations. Show that if T_1 and T_2 are both reflections, then T_1T_2 is a rotation.

15. If $\vec{A}$ and $\vec{B}$ are linearly dependent vectors then prove that $\vec{A} \times \vec{B} = \hat{0}$.

16. In terms of coordinates of $X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$, calculate the images of R^2_π and $R^3_{\frac{\pi}{4}}$.

17. If S is reflection in the x_3 – axis and T is projection to the x_3 – axis, then show that $m(S) = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

18. Let $m = \begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix}$ then find me^s_{21} and $p_{13}m$.

19. Let π be a plane and let P be projection on the plane π. Show that P does not have an inverse.

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20. Find all eigenvalues of the matrix $\begin{pmatrix} 5 & 6 \\ 3 & -2 \end{pmatrix}$.

21. Describe the quadric surface $x^2 + 2xy - 2xz = 1$.

22. If S is an isometry, then show that S^{-1} is also an isometry.

SECTION – III

Answer **any 6** questions from this Section. **Each** question carries **4** marks.

23. Let A be a linear transformation with matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ such that $ad - bc \neq 0$. If π is any parallelogram with one vertex at 0, then prove that

area $(A(\pi))$ = (absolute value of $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$) area (π).

24. Let B be a transformation of the plane such that $B(X + \bar{X}) = B(X) + B(\bar{X})$ and $B(tX) = tB(x)$ where X and $\bar{X}$ are vectors and t is a scalar. Show that B is linear.

25. Find the inverse of $R_{\frac{\pi}{2}}$.

26. Let T be a linear transformation of R^3. Prove that T has an inverse if and only if the equation $T(X) = Y$ has one and only solution for each Y.

27. Find all solution of the non homogeneous system

$x + 2y + 3z = 1$

$x + 3y + 6z = 3$

$2x + 6y + 13z = 5$

28. Prove that if T is an isometry of R^3 then $\det(T) = \pm 1$.

29. Let m be a symmetric 3×3 matrix and let M be the corresponding linear transformation. Let t_1, t_2, t_3 denote the eigenvalues of M and set $d = \begin{pmatrix} t_1 & 0 & 0 \\ 0 & t_2 & 0 \\ 0 & 0 & t_3 \end{pmatrix}$.

Prove that there exist an orthogonal matrix r of R^3 such that $m = rdr^{-1}$.

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30. Show that the four vectors $\begin{pmatrix} x \\ y \\ u \\ v \end{pmatrix}, \begin{pmatrix} -y \\ x \\ -v \\ u \end{pmatrix}, \begin{pmatrix} u \\ -v \\ -x \\ y \end{pmatrix}, \begin{pmatrix} v \\ u \\ -y \\ -x \end{pmatrix}$ are mutually orthogonal and all have the same length.

31. Find a cubic curve with the equation $y = ax^3 + bx^2 + cx + d$ passing through the four points (1, 0), (2, 2), (3, 12), (4, 36).

SECTION – IV

Answer **any 2** questions from this Section. **Each** question carries **15** marks.

32. Let $A = \begin{pmatrix} 3 & -4 \\ 2 & -6 \end{pmatrix}$. Find all eigenvalues and corresponding eigenvectors. Also find the matrices P and D such that $P^{-1}AP = D$ is diagonal.

33. Let m be a symmetric 3×3 matrix and let M be the corresponding linear transformation. Prove that there exists three orthogonal unit vectors X_1, X_2, X_3 such that each X_i is an eigenvector of M.

34. a) Let T be a linear transformation of R^3 denote by A, B, C the row vectors of the matrix of T. Prove that T has an inverse if and only if $A.(B \times C) \neq 0$.

    b) Solve the system

$$x_1 - 2x_2 + 3x_3 = 0$$
$$x_1 + x_2 + x_4 = 0$$
$$4x_3 - x_4 = 0$$

    c) Show that $\left(e_{32}^{s}\right)^{-1} = e_{32}^{-s}$.

35. a) Prove that a linear transformation A on R^3 preserves orientation if and only if $\det(m(A)) > 0$.

    b) Prove that if A is a linear transformation of R^3, each eigenspace of A is either a line through the origin or a plane through the origin or all of R^3.

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